On the Dutch Book Method for Conditionals

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World Sequence Day

May 24, 2024

Preliminary

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Matt tosses a six-sided fair die.

1. What is the probability that the die lands on a 2 if it lands on a prime?[1]

[1] The prime numbers are 2,3,5.

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1. What is the probability that the die lands on a 2 if it lands on a prime?[1] 1/3
2. What is the probability that the die lands on a 1 if it does not land on a prime?

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Preliminary

Question

What is the probability that either the die lands on 2 if it lands on a prime, or it lands on 1 if it doesn't land on a prime, i.e. what is $p(\text{prime} \to 2 \vee \neg\text{prime} \to 1)$?

Answer 1

It's $1/3$.[2]

[2] This is the answer according to Égré et al., manuscript

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Answer 1

It's 1/3.[2]

$$
\begin{aligned}
& p(\text{prime} \to 2 \vee \neg\text{prime} \to 1) \\
=& p(\text{prime} \to (1 \vee 2) \vee \neg\text{prime} \to (1 \vee 2)) \\
=& p(1 \vee 2) \\
=& \frac{1}{3}.
\end{aligned}
$$

[2] This is the answer according to Égré et al., manuscript

Answer 2

It's 5/9.[3]

[3] This is the answer according to e.g. Goldstein and Santorio, 2021; McGee, 1989; B. C. Van Fraassen, 1976.

Answer 2

It's 5/9.[3]

$$
\begin{aligned}
& p(\text{prime} \rightarrow 2 \vee \neg\text{prime} \rightarrow 1) \\
=& 1 - p(\text{prime} \rightarrow (3 \vee 5) \wedge \neg\text{prime} \rightarrow (4 \vee 6)) \\
=& 1 - p(\text{prime} \rightarrow (3 \vee 5))p(\neg\text{prime} \rightarrow (4 \vee 6)) \\
=& 1 - \frac{2}{3} \cdot \frac{2}{3} \\
=& \frac{5}{9}
\end{aligned}
$$

[3] This is the answer according to e.g. Goldstein and Santorio, 2021; McGee, 1989; B. C. Van Fraassen, 1976.

Preliminary

Answer 1

It's 1.

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If you learn that the die lands even:

$$p_{\text{even}}(\text{prime} \rightarrow 2 \vee \neg\text{prime} \rightarrow 1) \geq p_{\text{even}}(\text{prime} \rightarrow 2) = 1$$

Preliminary

Answer 1

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If you learn that the die lands even:

$$p_{\text{even}}(\text{prime} \rightarrow 2 \vee \neg\text{prime} \rightarrow 1) \geq p_{\text{even}}(\text{prime} \rightarrow 2) = 1$$

If you learn that the die lands odd:

$$p_{\text{odd}}(\text{prime} \rightarrow 2 \vee \neg\text{prime} \rightarrow 1) \geq p_{\text{odd}}(\neg\text{prime} \rightarrow 1) = 1$$

Preliminary

Answer 1

It's 1.

If you learn that the die lands even:

$$p_{\text{even}}(\text{prime} \rightarrow 2 \vee \neg\text{prime} \rightarrow 1) \geq p_{\text{even}}(\text{prime} \rightarrow 2) = 1$$

If you learn that the die lands odd:

$$p_{\text{odd}}(\text{prime} \rightarrow 2 \vee \neg\text{prime} \rightarrow 1) \geq p_{\text{odd}}(\neg\text{prime} \rightarrow 1) = 1$$

So you will become certain of the disjunction no matter what you learn. By Reflection (B. C. Van Fraassen, 1995; C. Van Fraassen, 1984), you should be certain of it now.

There are two urns, X and Y.[4]

- Urn X: 8 red balls, 2 blue balls, 0 spotted
- Urn Y: 2 red balls, 8 blue balls, all spotted

Stefan draws a ball from either of the two urns, and he flips a fair coin to decide which urn to draw from.

[4] Cf. (Kaufmann, 2004, 2009; Khoo, 2016). I borrowed this version of the example from Mandelkern (manuscript).

Preliminary

Question

What is the probability that if the ball drawn by Stefan is red, then it is spotted?

Answer 1

It's $1/2$.

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It's $1/2$.

$$p(R \rightarrow S) = p(X) = \frac{1}{2}.$$

Preliminary

Answer 1

It's 1/2.

$$p(R \to S) = p(X) = \frac{1}{2}.$$

Answer 2

It's 4/5.

Preliminary

Answer 1

It's 1/2.

$$p(R \to S) = p(X) = \frac{1}{2}.$$

Answer 2

It's 4/5.

$$p(R \to S) = p(S|R) = \frac{8}{10} = 0.8$$

Main Question

For these questions, is there one single correct answer, or are some/all the answers proposed admissible?

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How do we decide?

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- Simpson's paradox

Proposal

1. A probability judgment is irrational if it is Dutch-bookable, i.e. licenses accepting a set of bets that are individually fair but jointly guarantee a sure loss.

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1. A probability judgment is irrational if it is Dutch-bookable, i.e. licenses accepting a set of bets that are individually fair but jointly guarantee a sure loss.
2. There are different notions of Dutch books (fair bets), which vindicate different (and possibly incompatible) probability judgments about conditionals.

Plan

Formal setup

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Let At be a set of propositional atoms and $\mathcal{L}_C$ a conditional language generated by the grammar

$$\mathcal{L}_C \coloneqq \alpha \mid \neg p \mid p \wedge p \mid p \vee p \mid p \rightarrow p$$

[5] All fragments are closed under $\neg, \vee$ and $\wedge$

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$$\mathcal{L}_C := \alpha \mid \neg p \mid p \wedge p \mid p \vee p \mid p \rightarrow p$$

Let

- $\mathcal{L}_B$: the Boolean fragment of $\mathcal{L}$ (no conditionals)
- $\mathcal{L}_S$: the fragment that only contains simple conditionals;
- $\mathcal{L}_R$: the fragment that contains no left-nested conditionals.[5]

[5] All fragments are closed under $\neg$, $\vee$ and $\wedge$

Formal setup

Let $\mathcal{L} \in \{\mathcal{L}_B, \mathcal{L}_S, \mathcal{L}_R, \mathcal{L}_C\}$.

A **credence function over** $\mathcal{L}$ is a function $c : \mathcal{L} \to [0, 1]$.[6]

[6] I take the objects of credences to be sentences rather than propositions in order to stay neutral on the logic of conditionals and whether they express propositions.

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A **credence function over** $\mathcal{L}$ is a function $c : \mathcal{L} \to [0, 1]$.[6]

What does it mean for c to be **Dutch-bookable**?

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Formal setup

Suppose $c : \mathcal{L}_B \to [0, 1]$ and $c(A \vee \neg A) = 1$ but $c(A) = c(\neg A) = 0$.

[7]Throughout I assume that the agent values money linearly.

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Suppose $c : \mathcal{L}_B \to [0, 1]$ and $c(A \vee \neg A) = 1$ but $c(A) = c(\neg A) = 0$.

- Bet 1: buy a \$1 bet on $A \vee \neg A$ at \$1.

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- Bet 3: sell a \$1 bet on $\neg A$ at \$0.[7]

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	A	$\neg A$
Bet 1	0	0
Bet 2	-1	0
Bet 3	0	-1
Net	-1	-1

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Formal setup

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Bet 3	0	-1
Net	-1	-1

This payoff table assumes that the bets are settled "classically":

- the agent either wins or loses her bets on A and $\neg A$ (e.g. it never gets called off);
- the agent loses her bet on A iff she wins her bet on $\neg A$

Formal setup

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Bet 2	-1	0	0
Bet 3	0	-1	0
Net	-1	-1	0

The agent doesn't suffer "sure" loss (i.e. loss at all informational states).

Formal setup

We represent a way of settling bets on $\mathcal{L}$ by a **settlement function** $s : \mathcal{L} \to [0, 1]$.

$s(p) = x$: the seller pays the buyer \$$x$ for a unit bet on p according to s.

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Definition

c is **Dutch-bookable relative to** $\mathcal{S}$ if there are $\alpha_1, \ldots, \alpha_n \in \mathbb{R}, p_1, \ldots, p_n \in \mathcal{L}$ such that

$$\sum_{i=1}^{n} \alpha_i (c(p_i) - s(p_i)) < 0, \forall s \in \mathcal{S}$$

Formal setup

Example

Let $\mathcal{L} = \mathcal{L}_B$. We say $s : \mathcal{L}_B \to [0,1]$ is **Tarskian** if $s(p) \in \{0,1\}$ and

- $s(\neg p) = 1$ iff $s(p) = 0$;
- $s(p \wedge q) = 1$ iff $s(p) = 1$ and $s(q) = 1$;
- $s(p \vee q) = 1$ iff $s(p) = 1$ or $s(q) = 1$

Formal setup

Fact (De Finetti, 2017 (1972))

Let $\mathcal{S}$ be the set of Tarskian settlement functions on $\mathcal{L}_B$. Then c is not Dutch-bookable relative to $\mathcal{S}$ iff c is a classical finitely additive probability function.

Formal setup

Example

Let $\mathcal{L} = \mathcal{L}_B$. Say $p \vDash q$ if $s(q) = 1$ whenever $s(p) = 1$ for any Tarskian settlement function on $\mathcal{L}_B$. The set of **DS-settlement functions** is the set $\mathcal{S} = \{s_p : p \in \mathcal{L}_B, \nvDash \neg p\}$ where

$$s_p(q) = \begin{cases} 1 & \text{if } p \vDash q \\ 0 & \text{otherwise} \end{cases}$$

Formal setup

Fact (Jaffray, 1989)

Let $\mathcal{S}$ *be the set of Tarskian settlement functions on* $\mathcal{L}_B$. *Then* c *is not Dutch-bookable relative to* $\mathcal{S}$ *iff* c *is a classical Dempster-Shafer belief function.*

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To me, this result suggests that Dempster-Shafer belief functions are not competitors to Bayesian probabilities; they model different kinds of objects (cf. Paris, 2001).

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Goal: Generalize these results to languages involving conditionals.

Main result and applications

Main result

Theorem

Fix $\mathcal{L}$. Let $\mathcal{S}$ be a finite[8] *set of settlement functions for $\mathcal{L}$. c is not Dutch-bookable relative to $\mathcal{S}$ iff there exists a probability function π over $\mathcal{S}$ such that for all $p \in \mathcal{L}$,*

$$c(p) = \sum_{s \in \mathcal{S}} \pi(s) s(p)$$

[8] I assume finiteness to sidestep issues involving probabilities over infinite sets, e.g. measurability.

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Proof sketch. Hyperplane separation theorem.

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Main result

This representation result allows us to connect properties of settlement functions with properties of credences invulnerable to Dutch-books

Corollaries

Suppose c is not Dutch-bookable relative to $\mathcal{S}$. If

- for every $s \in \mathcal{S}$, s satisfies property Φ,

[9] For some results we also have the converse, which is often the more interesting direction, but I'll focus on this direction first because it's often easier to state.

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This representation result allows us to connect properties of settlement functions with properties of credences invulnerable to Dutch-books

Corollaries

Suppose c is not Dutch-bookable relative to $\mathcal{S}$. If

- for every $s \in \mathcal{S}$, s satisfies property Φ,
- then c satisfies property Ψ.[9]

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Application I: Stalnaker's Thesis

Let $\mathcal{L} = \mathcal{L}_S$. Fix $c : \mathcal{L} \to [0,1]$. Let $\mathcal{S}$ be a set of settlement functions on $\mathcal{L}$ that are Tarskian when restricted to $\mathcal{L}_B$. Suppose c is not Dutch-bookable relative to $\mathcal{S}$.

Fact

If for all $s \in \mathcal{S}$,

$$s(A \to B) = \begin{cases} s(B) & \text{if } s(A) = 1 \\ c(A \to B) & \text{if } s(A) = 0. \end{cases}$$

Then $c(A)c(A \to B) = c(A \wedge B)$.

Application I: Stalnaker's Thesis

Proof.

$$
\begin{aligned}
c(A \to B) &= \sum_s \pi(s) s(A \to B) \\
&= \sum_{s:s(A)=1, s(B)=1} \pi(s) + \sum_{s:s(A)=0} \pi(s) c(A \to B) \\
&= c(A \wedge B) + c(\neg A) c(A \to B)
\end{aligned}
$$

Application I: Stalnaker's Thesis

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So

$$c(A) c(A \to B) = c(A \wedge B)$$

$\square$

Application II: Local Conditioning

Let $\mathcal{X} = \{X_1, \dots, X_n\} \subseteq \mathcal{L}_B$ be a partition relative to $\mathcal{S}$, i.e. for all $s \in \mathcal{S}$, $s(\bigvee_i X_i) = 1$ and $s(X_i \wedge X_j) = 0$ for all $i \neq j$.

Fact

If for all $s \in \mathcal{S}$ and all i, if $s(X_i) = 1$, then

$$s(A \to B) = s(AX_i \to B) = \begin{cases} s(B) & \text{if } s(A) = 1 \\ c(AX_i \to B) & \text{otherwise} \end{cases}$$

Then $c(A \to B) = \sum_i c(B|AX_i)c(X_i)$.

Application II: Local Conditioning

Compare:

- Stalnaker's Thesis (Global Conditioning): if $s(X_i) = 1$ and $s(A) = 0$, then $s(A \to B) = \mathbf{c(A \to B)}$;
- Local Conditioning: if $s(X_i) = 1$ and $s(A) = 0$, then $s(A \to B) = \mathbf{c(AX_i \to B)}$

Application II: Local Conditioning

There are two urns, X and Y.

- Urn X: 8 red balls, 2 blue balls, 0 spotted
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Presumably, $c(XR \to S) = 1$ and $c(YR \to S) = 0$.

	XR	XB	YR	YB
Conforming	0	$\frac{4}{5}$	1	$\frac{4}{5}$
Non-conforming	0	0	1	1

Application II: Local Conditioning

As Mandelkern (manuscript) points out, we can get non-conforming judgments even when there are no salient partitions.

Suppose Stephen tosses a fair coin. If the coin lands heads, he will draw a ball at random from one of the two urns. If the coin lands tails, he'll go have lunch.

Application II: Local Conditioning

As Mandelkern (manuscript) points out, we can get non-conforming judgments even when there are no salient partitions.

Suppose Stephen tosses a fair coin. If the coin lands heads, he will draw a ball at random from one of the two urns. If the coin lands tails, he'll go have lunch.

Intuitively, $p(R \rightarrow S) = \frac{1}{2}$, but, assuming there is $s \in \mathcal{S}$ such that $s(T) = 1$, $\{X, Y\}$ no longer forms a partition relative to $\mathcal{S}$.

Application II: Local Conditioning

Let $\mathcal{X} = \{X_0, X_1, \ldots, X_n\} \subseteq \mathcal{L}_B$ be a partition relative to $\mathcal{S}$.
Suppose for all $s \in \mathcal{S}$ and all i,

- if $s(X_0) = 1$, then $s(A \to B) = \begin{cases} s(B) & \text{if } s(A) = 1 \\ c(A \to B) & \text{otherwise} \end{cases}$
- for $i \geq 1$, if $s(X_i) = 1$, then

$$s(A \to B) = s(AX_i \to B) = \begin{cases} s(B) & \text{if } s(A) = 1 \\ c(AX_i \to B) & \text{otherwise} \end{cases}$$

Let $X = \bigvee_{i \geq 1} X_i$.

Application II: Local Conditioning

Then

$$c(A \to B) = \frac{c(ABX_0) + \sum_{i \geq 1} c(B|AX_i)c(X_i))}{c(X \vee A)}$$

In particular, if $A \vDash X$, then

$$c(A \to B) = \sum_{i \geq 1} c(B|AX_i)c(X_i|X).$$

Application II: Local Conditioning

	XR	XB	YR	YB	T
Conforming	0	$\frac{4}{5}$	1	$\frac{4}{5}$	$\frac{4}{5}$
Non-conforming	0	0	1	1	$\frac{1}{2}$

Application III: Trivalent Probability

Fix $c : \mathcal{L} \to [0, 1]$. $W \subseteq \{w : \mathcal{L} \to \{0, 1/2, 1\}$. For each w, define $s_w : \mathcal{L} \to [0, 1]$ as

$$s_w(p) = \begin{cases} w(p) & \text{if } w(p) \neq 1/2 \\ c(p) & \text{otherwise.} \end{cases}$$

Let $p_T = \{w \in W : w(p) = 1\}$ and $p_F = \{w \in W : w(p) = 0\}$.

Application III: Trivalent Probability

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Let $p_T = \{w \in W : w(p) = 1\}$ and $p_F = \{w \in W : w(p) = 0\}$.

Fact

c is not Dutch-bookable relative to $\mathcal{S} = \{s_w : w \in W\}$ iff there is a probability function $\pi \in \Delta(W)$ such that

$$c(p) = \frac{\pi(p_T)}{\pi(p_T) + \pi(p_F)}$$

Application IV: Sequence Probabilities

One of the defining properties of probabilities in Bernoulli models is that they satisfy **Independence**: if H, A are incompatible, then $c(H \wedge A \to B) = c(H)c(A \to B)$.

Fact

Suppose for all $s \in \mathcal{S}$,

- *Additivity.* $s(p) = s(A \wedge p) + s(\neg A \wedge p)$.
- *Conjunction.* $s(p \wedge q) = 0$ *if* $s(p) = 0$ *or* $s(q) = 0$.
- *Weak Cancellation. If* $s(A) = 0$, *then* $s(A \to B) = c(A \to B)$.

Then c satisfies Independence.

Application IV: Sequence Probabilities

In general, settlement functions defined in terms of trivalent semantics satisfy Conjunction and Weak Cancellation, but violate Additivity.

E.g. In the theory of Egre et al., if $s(H) = 1$ and H, A are incompatible, then

- $s(A \rightarrow B) = c(A \rightarrow B)$;
- $s(H \wedge A \rightarrow B) = 1$.

Application V: Back to the die

Question

What is the probability that either the die lands on 2 if it lands on a prime, or it lands on 1 if it doesn't land on a prime?

	1	2	3	4	5	6
$\mathbf{\frac{1}{3}}$	1	1	0	0	0	0
$\mathbf{\frac{5}{9}}$	1	1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\mathbf{1}$	1	1	1	1	1	1

Conclusion

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2. There are different notions of Dutch books (fair bets), which vindicate different (and possibly incompatible) probability judgments about conditionals.

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- Representation theorem for Stalnaker's Thesis, Local Conditioning, Trivalent Probabilities and Independence in terms of properties of settlement functions

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- Representation theorem for Stalnaker's Thesis, Local Conditioning, Trivalent Probabilities and Independence in terms of properties of settlement functions
- Generalization of the set-up to diachronic Dutch-books (the Update Thesis vs. Conditionalization; cf. Fusco (2023) and McNamara and Zhang (manuscript))

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Open problems

- Other probabilistic principles; left-nested conditionals

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- What are people's intuitions about settlement conditions for compounds of conditionals/nested conditionals? Cf. (Politzer et al., 2010)

Conclusion

Open problems

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- Analogue results in terms of accuracy
- Unconditionals, the Reflection Principle and the logic of dominance reasoning
- What are people's intuitions about settlement conditions for compounds of conditionals/nested conditionals? Cf. (Politzer et al., 2010)

Whether or not I am right about anything, there is much more work to be done!