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## 1 Indicatives and Subjunctives

Lewis (1981b) begins:

> Some think that in (a suitable version of) Newcomb's problem, it is rational to take only one box…They are convinced by **indicative** conditionals: **if I take one box I will be a millionaire, but if I take both boxes I will not**…
>
> Others, and I for one, think it rational to take both boxes…We are convinced by **counterfactual** conditionals: **If I took only one box, I would be poorer by a thousand than I will be after taking both.**

## 2 Some Formalism

- **Expected Utility.** Lewis claimed that …

$$EEU(A) = \sum_S Pr(S \mid A)Val(AS) \tag{1}$$

$$CEU(A) = \sum_S Pr(A >_{\mathrm{s}} S)Val(AS) \tag{2}$$

- **Stalnaker's Thesis.**

$$Pr(A >_{\mathrm{i}} S) = Pr(S \mid A) \tag{3}$$

- **Skyrms's Thesis.**[1]

$$Pr(A >_{\mathrm{s}} S) = \mathbb{E}_{Pr}(Ch(S \mid A)) \tag{4}$$

- **Principal Principle (PP).** *Suppose you expect to receive no inadmissible information and that your occurrent justified prior is $Pr$. Then:*

$$Pr(S \mid (Ch = \pi)) = \pi(S) \tag{PP}$$

$$Pr(S) = \mathbb{E}_{Pr}(Ch(S)) \tag{5}$$

$$= \sum_\pi Pr(Ch = \pi)\pi(S) \tag{6}$$

- **Combination of PP with (3) and (4).**

$$CEU(A) = \sum_S \mathbb{E}_{Pr}(Ch(S \mid A))Val(AS) \tag{7}$$

$$= \sum_S \mathbb{E}_{Pr}\left(\frac{Ch(AS)}{Ch(A)}\right)Val(AS) \tag{8}$$

*…The expectation of a ratio*

$$EEU(A) = \sum_S \left(\frac{Pr(AS)}{Pr(A)}\right)Val(AS) \tag{9}$$

$$= \sum_S \left(\frac{\mathbb{E}_{Pr}(Ch(AS))}{\mathbb{E}_{Pr}(Ch(A))}\right)Val(AS) \tag{10}$$

*… A ratio of expectations*

- **Bayesian Lore.** You are rationally required to update by conditionalization—viz., by the ratio of expectations.

## 3 Examples

- (SHOE BETS.)

[1] Skyrms (1981) and Skyrms (1984, Ch. 5).

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| | Bet 1 | Bet 2 | posterior probability if $\overline{p}$ | payoffs if $\overline{p}$ |
|---|---|---|---|---|
| $p$ | \$1 | $-$\$10 | 0 | $-$\$0.65 |
| $\overline{p}$ | \$0 | \$0 | 1 | \$6 |
| premium | $-$\$0.65 | \$6 | | Expectation: \$(6.00 − .65) = \$5.35 |

Table: for (SHOE BETS).

- **De Finetti payoffs**: you will pay premium of $\$Pr(B \mid A)$ for a bet which
  - pays \$1 if $(A \wedge B)$;
  - pays \$0 if $(A \wedge \overline{B})$
  - is called-off (premium refund) if $\neg A$.

$$k = [Pr(AB) \times 1 + Pr(A\bar{B}) \times 0] + [Pr(\bar{A}) \times k] \qquad \text{(DF)}$$

- (BIASED COINS.) You know two coins, $A$ and $B$, come from the same heavily biased coin factory. Their bias is either .9 towards heads or .9 away from heads: $Ch(A_H) = Ch(B_H) \in \{.1, .9\}$. Their flips, of course, are independent: $Ch(A_H \mid B_H) = Ch(A_H)$ and vice-versa.

  You're indifferent as to which way the coins are biased: $.5 = Pr(Ch(A_H) = .9) = Pr(Ch(B_H) = .9)$. It follows that: (i) $Pr(B_H \mid A_H) = .82$; (ii) $\mathbb{E}_{Pr}(Ch(B_H \mid A_H)) = .5$. So by Stalnaker's Thesis, $Pr(A_H >_i B_H) = .82$; by Skyrms's Thesis, $Pr(A_H >_s B_H) = .5$.[2]

  Coin $A$ is in your hand. Coin $B$ is about to flipped by nature.

I purchase from Penurious Paul, for \$0.65, a DeFinetti bet on $(A_H > B_H)$ at 65-35 odds for a stake of \$1 ... But then Wealthy William comes along and is willing to buy, *from me*, a DeFinetti bet on $(A_H > B_H)$ at 60-40 odds for a stake of \$100. *Combined: (i) \$-39.65 if $(A_H B_H)$; (ii) \$59.35 if $(A_H B_T)$; (iii) \$0 otherwise.*

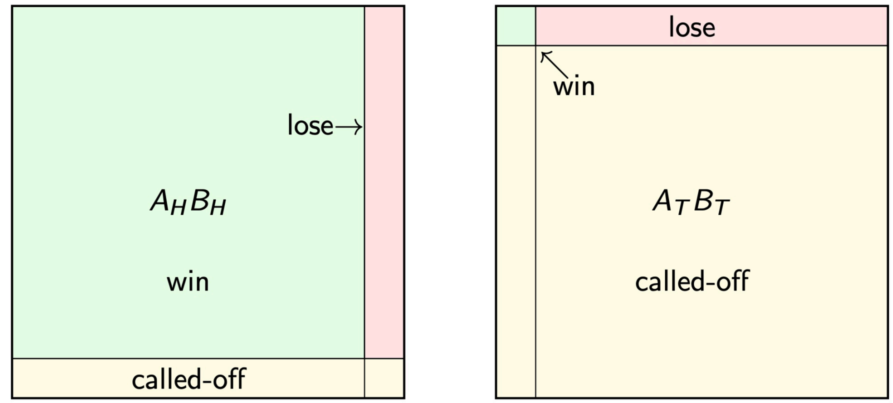


Figure 2: The $(A_H > B_H)$ Prior.

| | Bet 1 | Bet 2 | Posterior if **place heads** | payoffs if **place heads** |
|---|---|---|---|---|
| $A_H B_H$ | \$1 | $-$\$100 | .5 | $1 - 100 - 0.65 + 60 = -39.65$ |
| $A_H B_T$ | \$0 | \$0 | .5 | $0 + 0 - .65 + 60 = 59.35$ |
| $A_T$ | \$0.65 | $-$\$60 | 0 | |
| premium | $-$\$0.65 | \$60 | | Expectation: \$9.85 |

Table: for (BIASED COINS).

[2] The Skyrms's Thesis quantity, .5, is obvious. For Stalnaker: $Pr(A_H >_i B_H) = Pr(B_H \mid A_H) = \frac{Pr(A_H B_H)}{Pr(A_H)} = \frac{\sum_\pi Pr(Ch=\pi)\pi(A_H B_H)}{\sum_\pi Pr(Ch=\pi)\pi(A_H)} = \frac{.5(.9)^2+.5(.1)^2}{.5(.9)+.5(.1)} = \frac{.9^2+.1^2}{.9+.1} = \frac{.81+.01}{1} = .82$.

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| | | | | | |
|---|---|---|---|---|---|
| | $\pi^1(A_H) = \pi^1(B_H) = .9$; $\pi^1(A_H \mid B_H) = \pi^1(A_H)$; $\pi^1(B_H \mid A_H) = \pi^1(B_H)$ | | $\pi^2(A_H) = \pi^2(B_H) = .1$; $\pi^2(A_H \mid B_H) = \pi^2(A_H)$; $\pi^2(B_H \mid A_H) = \pi^2(B_H)$ | | |
| Place | $Ch_{A_H}(A_H \wedge B_H) = .9$ | \$1 | $Ch_{A_H}(A_H \wedge B_H) = .1$ | \$1 | |
| $A_H$ | $Ch_{A_H}(A_H \wedge B_T) = .1$ | \$0 | $Ch_{A_H}(A_H \wedge B_T) = .9$ | \$0 | |
| | $Ch_{A_H}(A_T) = 0$ | $\$k$ | $Ch_{A_H}(A_T) = 0$ | $\$k$ | = .5 |
| Flip | $Ch_\top(A_H \wedge B_H) = .81$ | \$1 | $Ch_\top(A_H \wedge B_H) = .01$ | \$1 | |
| | $Ch_\top(A_H \wedge B_T) = .09$ | \$0 | $Ch_\top(A_H \wedge B_T) = .09$ | \$0 | |
| | $Ch_\top(A_T) = .1$ | $\$k$ | $Ch_\top(A_T) = .9$ | $\$k$ | = .82 |
| Place | $Ch_{A_T}(A_H \wedge B_H) = 0$ | \$1 | $Ch_{A_T}(A_H \wedge B_H) = 0$ | \$1 | |
| $A_T$ | $Ch_{A_T}(A_H \wedge B_T) = 0$ | \$0 | $Ch_{A_T}(A_H \wedge B_T) = 0$ | \$0 | |
| | $Ch_{A_T}(A_T) = 1$ | $\$k$ | $Ch_{A_T}(A_T) = 1$ | $\$k$ | = undefined |

Figure 3: The $(A_H > B_H)$ Matrix.

A **valuated chance space for** $\mathcal{L}$ is a tuple $\langle W, \pi, V\rangle$, where (i) $\langle W, \pi\rangle$ is a chance space and (ii) $V$: prop $\to \wp(W)$ is a valuation function such that for every $w \in W$, there is some sentence $\phi_w$ such that $V(\phi_w) = \{w\}$. The truth-conditions for $\phi \in \mathcal{L}$ are relativized to $w \in W$ and $\pi$-pairs (extending Kocurek, 2022):

- $\pi, w \Vdash p$ iff $w \in V(p)$
- $\pi, w \Vdash \neg\phi$ iff $\pi, w \nVdash \phi$
- $\pi, w \Vdash (\phi \wedge \psi)$ iff $\pi, w \Vdash \phi$ and $\pi, w \Vdash \psi$
- $\pi, w \Vdash (Ch = \pi')$ iff $\pi = \pi'$
- $\pi, w \Vdash (Ch(\phi) = n)$ iff $\pi(\llbracket\phi\rrbracket_\pi) = n$, where $\llbracket\phi\rrbracket_\pi := \{w' \in W \mid \pi, w' \Vdash \phi\}$

A **Principal Principle (PP)-Compliant Probability Model** on $W$ is a tuple $\mathcal{M} = \langle Pr, \Pi, W, V\rangle$, where

1. $\Pi$ is a set of chance hypotheses $\pi^1, \pi^2 \ldots \pi^n$ such that for each $\pi^i$, $\langle W, \pi^i, V\rangle$ is a valuated chance space over $W$.
2. $Pr$ is a probability distribution over $\pi \in \Pi$. When $Pr(\pi) = n$ for some $\pi \in \Pi$ and $n \in [0,1]$, we say equivalently that $Pr(Ch = \pi) = n$.
3. For all $\phi \in \mathcal{L}$:
   $Pr(\phi \mid Ch = \pi) = \pi(\llbracket\phi\rrbracket_\pi)$, which entails

$$Pr(\phi) = \sum_\pi Pr(Ch = \pi)\pi(\llbracket\phi\rrbracket_\pi)$$

A **decision problem** $D = \langle \mathcal{M}, \boldsymbol{A}, \boldsymbol{S}, Val\rangle$ pairs a PP-Compliant Probability Model $\mathcal{M}$ with

1. an orthogonal partitioning of $W$ into *acts* $\boldsymbol{A}$ and states $\boldsymbol{S}$. Outcomes $O$ are members of $\boldsymbol{A} \times \boldsymbol{S}$.
2. a utility function $Val$: $O \mapsto \mathbb{R} \cup \{\mathbb{R}\}$.[a] $\forall w', w'' \in O$, we say that $Val(w') = Val(w) = Val(O)$.
3. a set of *moves* $\boldsymbol{M}$ such that (i) $\boldsymbol{A} \subset \boldsymbol{M}$; (ii) $\top \in \boldsymbol{M}$. For now we will consider the case where $\boldsymbol{M} = \boldsymbol{A} \cup \{\top\}$.

[a]This allows De Finetti-style conditional bets to take the value $\{\mathbb{R}\}$ in the premium-refund condition.

$$EU(M) = \sum_\pi \sum_{O \in \boldsymbol{A} \times \boldsymbol{S}} Pr(Ch = \pi)\pi_M(O)Val(O) \quad (12)$$

in sequence semantics, we can lift this to:

$$EU(M) = \sum_\pi \sum_{O \in \{A:\, A \cap M \neq \varnothing\} \times \boldsymbol{S}} Pr(Ch = \pi)\pi(M > O)Val(O) \quad (13)$$

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**Theorem 1.** If $M = A \in \boldsymbol{A}$, $EU(M) = CEU(A)$.

**Theorem 2.** If $M = \top$, then $EU(M) = EEU(M)$.

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